Numerical Simulations of the Jet in the Crab Nebula

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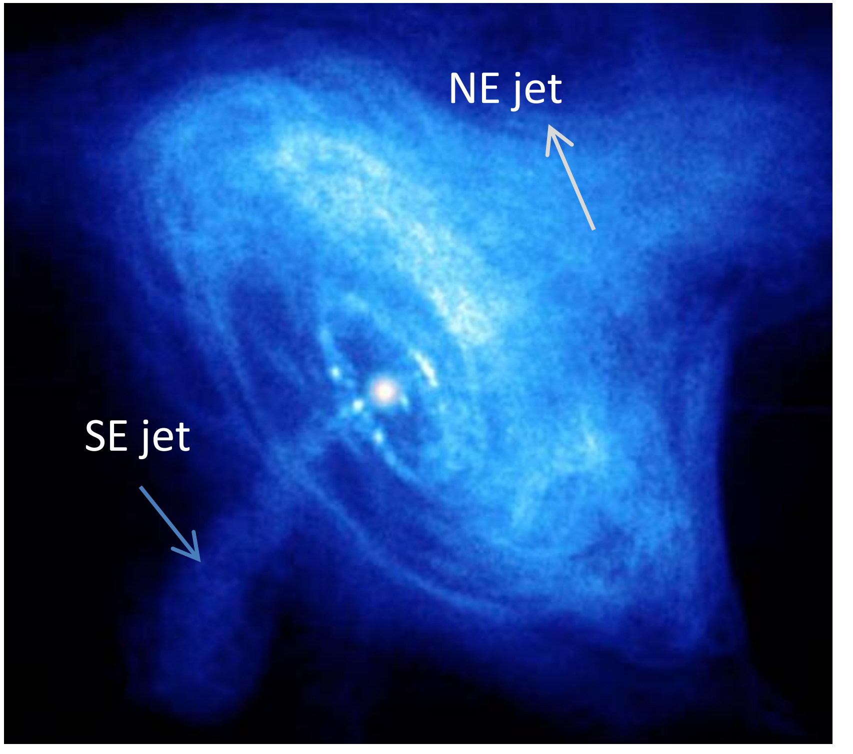
Outline

1. *Observational Evidence*
2. *Numerical Models of Relativistic MHD jets*
3. *Results*
4. *Summary*

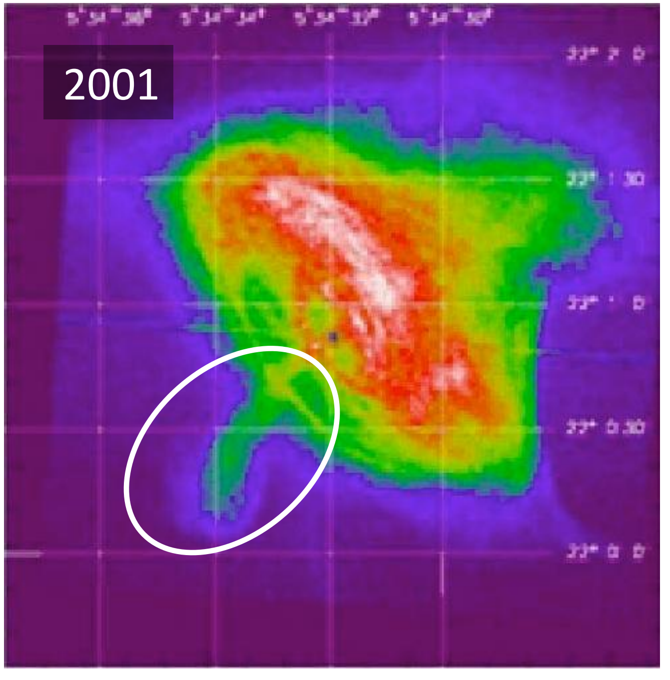
Observational Evidence

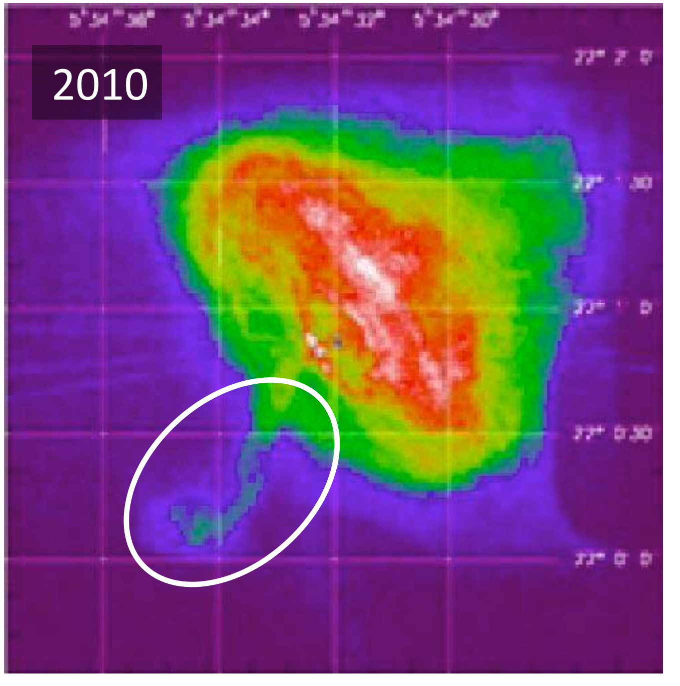


- X-ray observation (Chandra) show the emergence of a bipolar jets and extending to the SE and NEW of the pulsar;
- A region of diffuse emission (Anvil) may be associated with shocks and marks the base of the X-ray and optical jet;
- Knots of emission are seen along the jets;
- In the SE jet material flows with v/c~0.4 slowing down to ~0.02 into the nebula;

Jet Wiggling

- SE jet morphology is "S" shaped and show remarkable time variability:



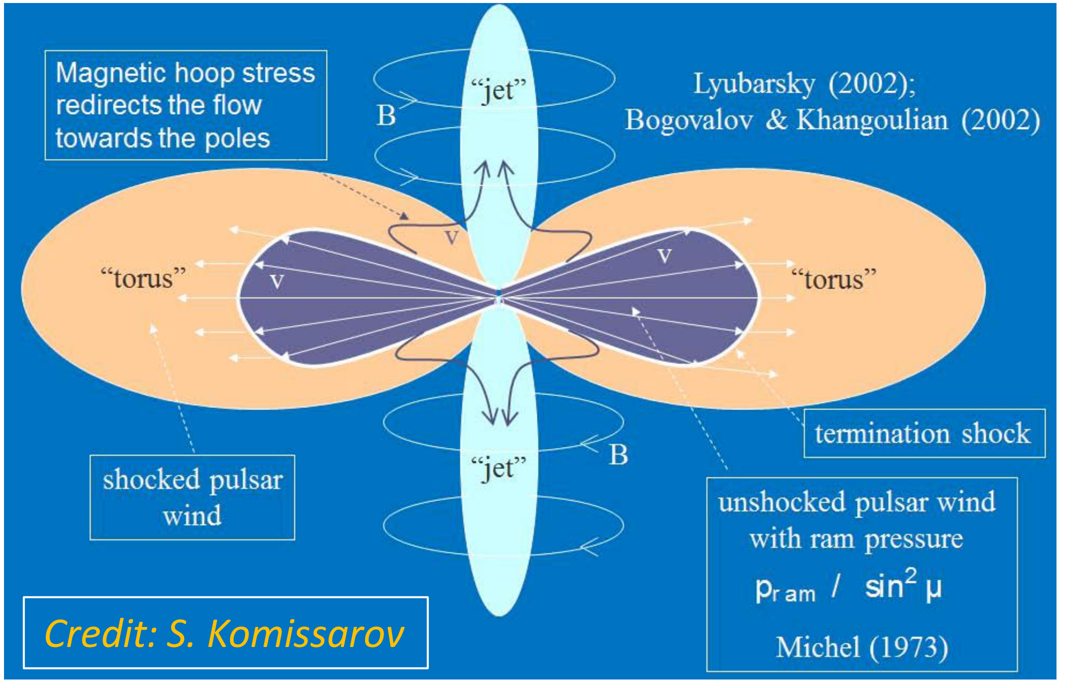


- → evidence for some kind of flow instability (Current Driven ?)

On the Origin of the Jet

- Jet forms downstream of the wind termination shock;

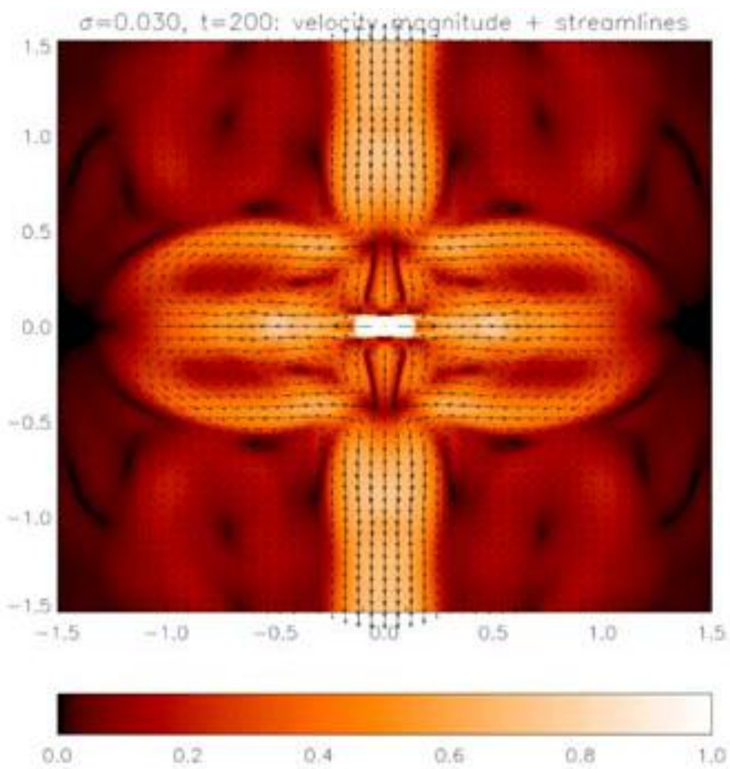
- Magnetic fields confine matter towards polar axis;
  → "***tooth-paste***" effect: hoop stress of the azimuthal magnetic field carried by the wind (Lyubarsky 2002).

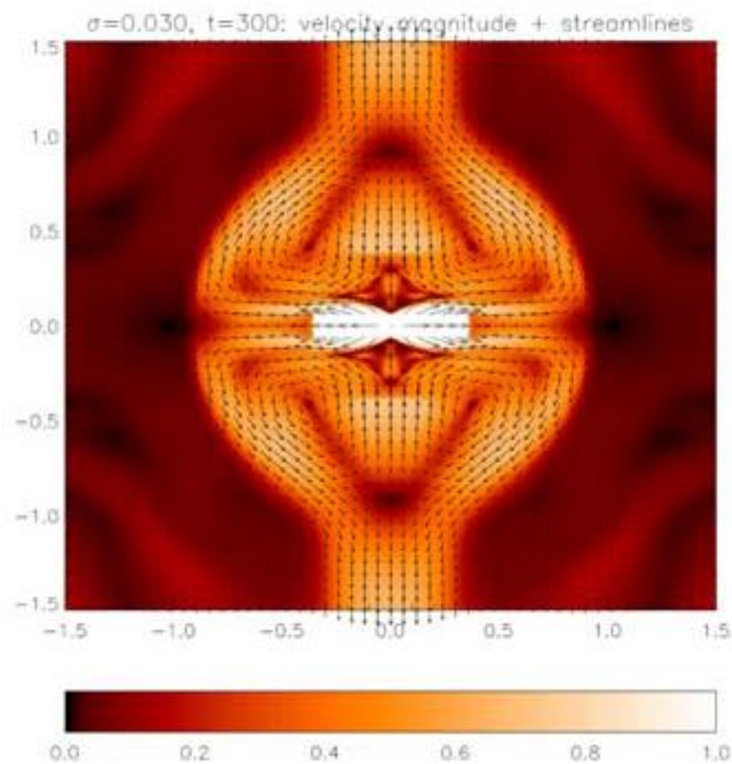


- Models confirmed by 2D axisymmetric numerical simulations (Komissarov & Lyubarski 2003,2004, Del Zanna et al. 2004, Bogovalov et al. 2005)

Jet Origin: previous results

- For moderate/large $\sigma = B^2/(8\pi\rho c^2\gamma^2)$ magnetic hoop stress suppresses high velocity outflows in the equatorial plane and divert them towards the polar axis partially driving the super-fast jet[1]

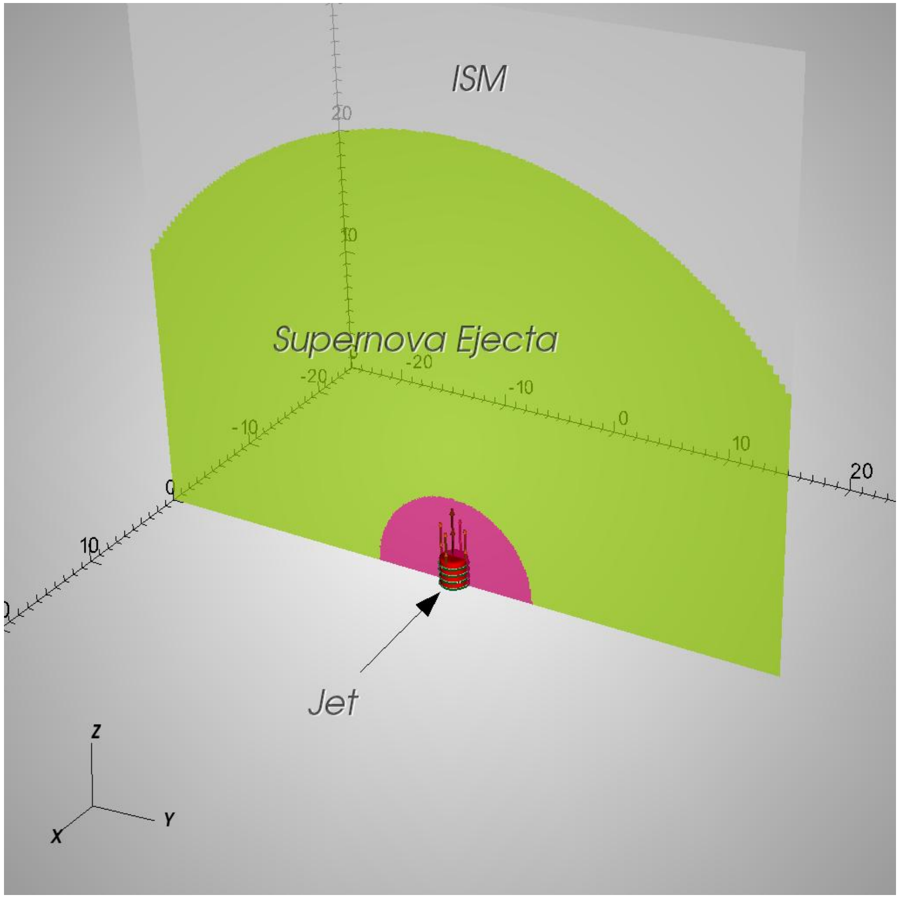




[1]Del Zanna et al, A&A (2004) 421,1063

3D Jet models

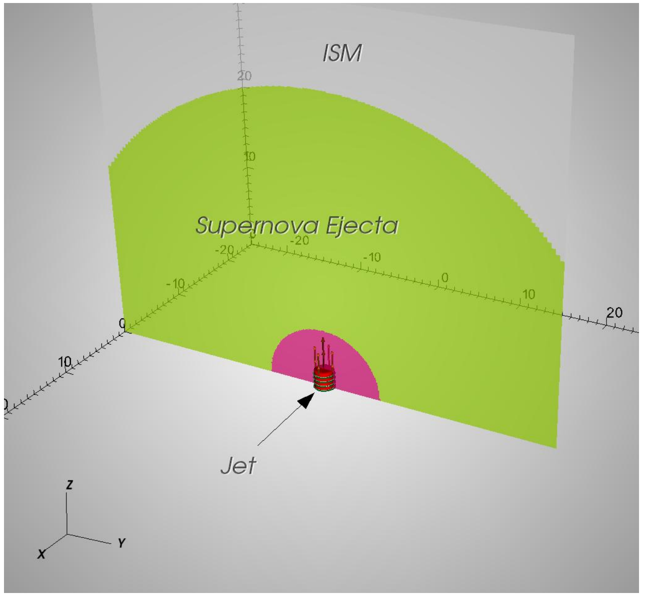
- Initial conditions from Del Zanna et al. (2004).
- Inside $0.2 < r < 1$ (ly): freely expanding supernova ejecta ($3\ M_{sun}$, $E = 10^{51}$ erg)
- Jet enters at the lower z boundary;
- Pulsar wind structure not considered: jet already formed as the result of the collimation process;
- Jet radius $R_j = 3 \cdot 10^{16}$ cm
- Computational domain:
  $x,y \in [-25,25]\ R_j/c$, $z \in [0, 80]\ R_j/c$;
  ($\approx 1.6 \times 2.5$ ly).



3D Jet models

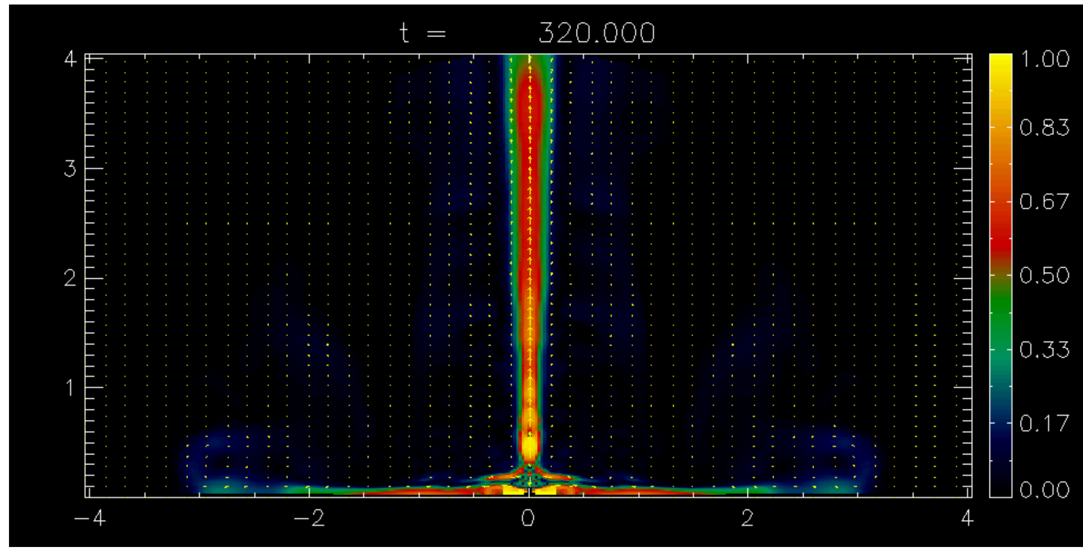
- Jet flow modeled by 5 parameters:

1. Sonic flow Mach number: $M_s = v_j / c_s$
2. Bulk Lorentz factor: $\gamma_j = (1-v^2_j)^{-1/2}$
3. Jet/ambient dens. contrast: $\eta=\rho_j / \rho_e$
4. Magnetization: $\sigma = B^2/(8\pi\rho\gamma^2)$;
5. Pitch angle: $P = RB_z / B_\phi$



Parameter Constraints

- Parameters are constrained by 2D axisymmetric results
  - $1.3 \lesssim Ms \lesssim 2$ → hot jet
  - $2 \lesssim \gamma \lesssim 4$
  - σ = ?
  - Density contrast $\eta \lesssim 10^{-6}$
  - Azimuthal field implies Pitch $\rightarrow 0$ (Bz = 0)



- This leaves γ and σ as free parameters.
- We consider hollow (η=10^{-6}), hot (M_s = 1.7) jets initially carrying purely axial current ($B_\phi \neq 0$, $B_z = B_R = 0$).
- $p(R)$ and $B_\phi(R)$ set by radial momentum balance across the jet

Equations

- We solve the equations for a relativistic perfectly conducting fluid describing energy/momentum and particle conservation (relativistic MHD equations)

$$\boxed{\begin{aligned}
\frac{\partial}{\partial t}(\rho\gamma)+\nabla\cdot(\rho\gamma\mathbf{v}) &= 0 \\
\frac{\partial \mathbf{m}}{\partial t}+\nabla\cdot\left[w\gamma^2\mathbf{v}\mathbf{v}-\mathbf{B}\mathbf{B}-\mathbf{E}\mathbf{E}\right]+\nabla p_t &= 0 \\
\frac{\partial \mathbf{B}}{\partial t}-\nabla\times(\mathbf{v}\times\mathbf{B}) &= 0 \\
\frac{\partial \mathcal{E}}{\partial t}+\nabla\cdot(\mathbf{m}-\rho\gamma\mathbf{v}) &= 0
\end{aligned}}$$

$$\boxed{\mathcal{E}=w\gamma^2-p+\frac{\mathbf{B}^2+\mathbf{E}^2}{2}-\rho\gamma}$$

- We use the PLUTO[1,2] code for astrophysical fluid dynamics (freely distributed http://plutocode.ph.unito.it)
- Numerical resolution 320 x 320 x 768 zones ($\approx$ 20 point on the jet)

Simulation Cases

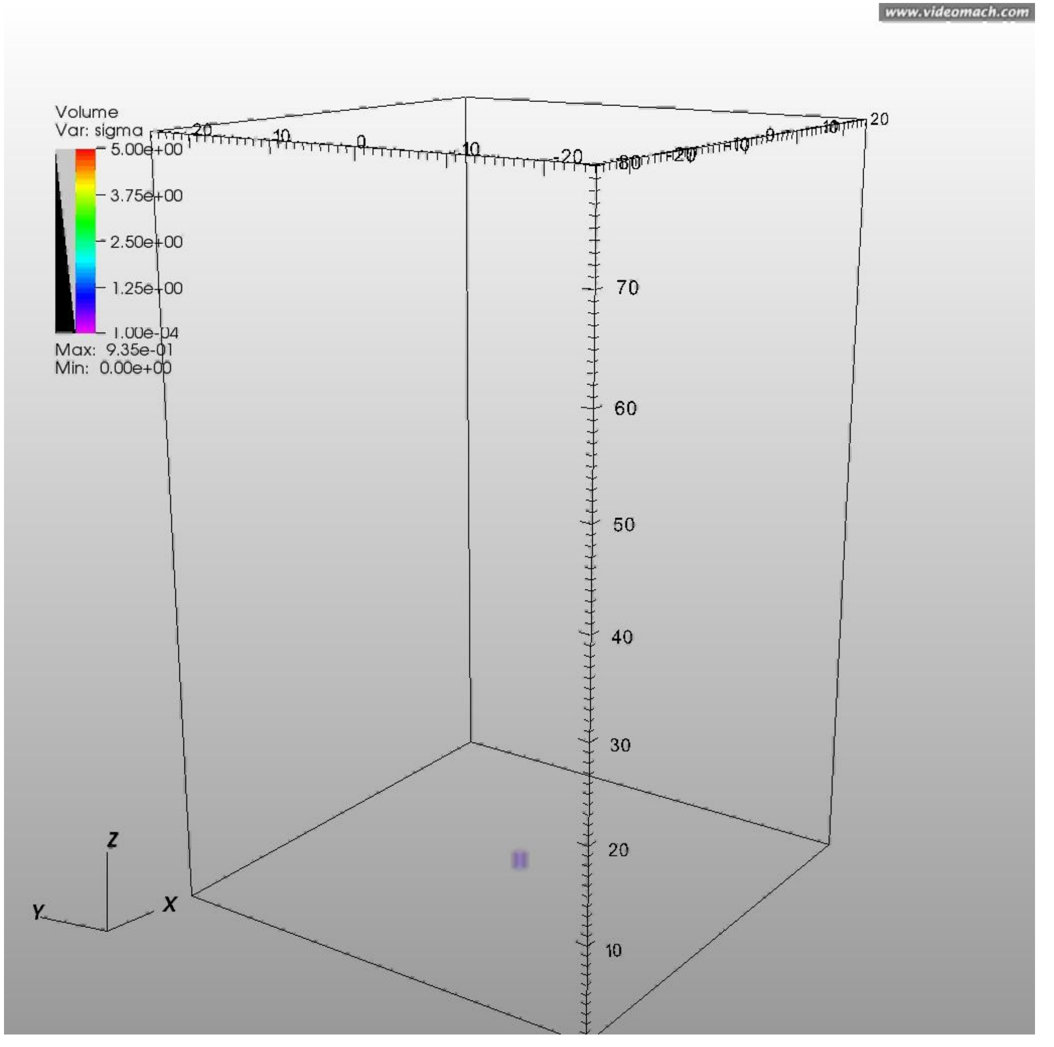
- We explore different values of Lorentz factor γ (= 2, 4) and magnetization σ (= 0.1, 1, 10) for a total of 6 different cases:

Case	γ	σ	*Plasma* β
A1	2	0.1	*4.5*
A2	2	1	*0.6*
A3	2	10	*0.2*
B1	4	0.1	*11.4*
B2	4	1	*1.2*
B3	4	10	*0.15*

Results: Case A2

Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

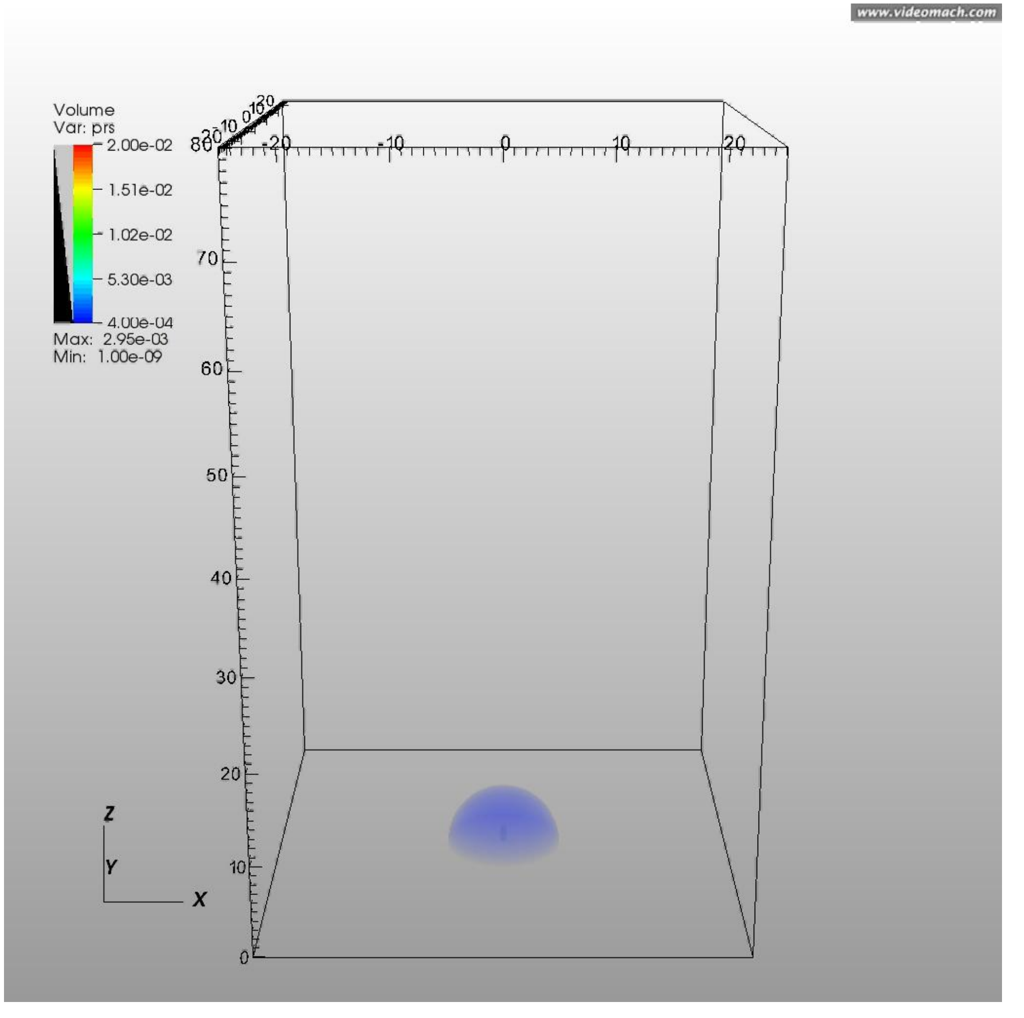
Sigma distribution



Results: Case A2

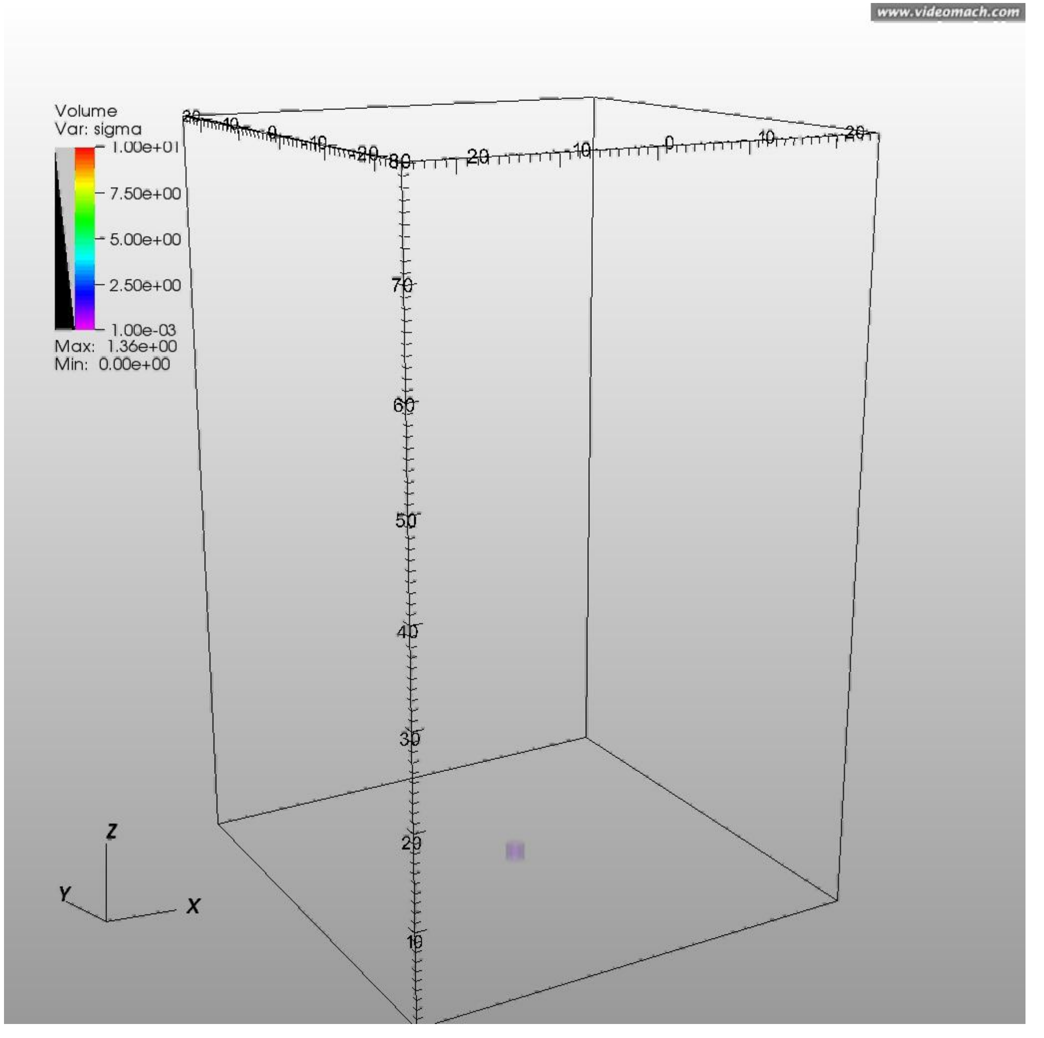
Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

Pressure distribution



Results: Case B2

Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

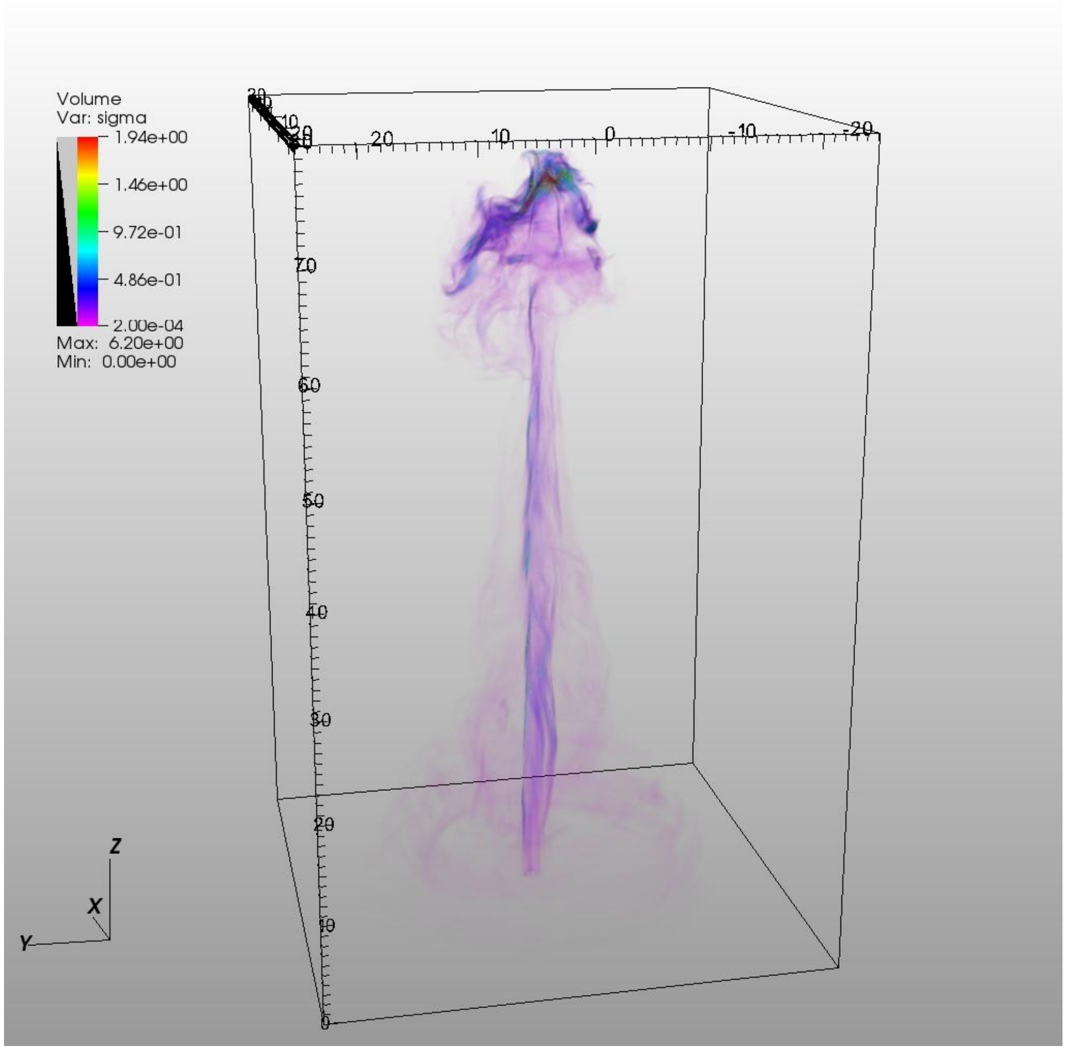


Sigma distribution

Results: Case B1

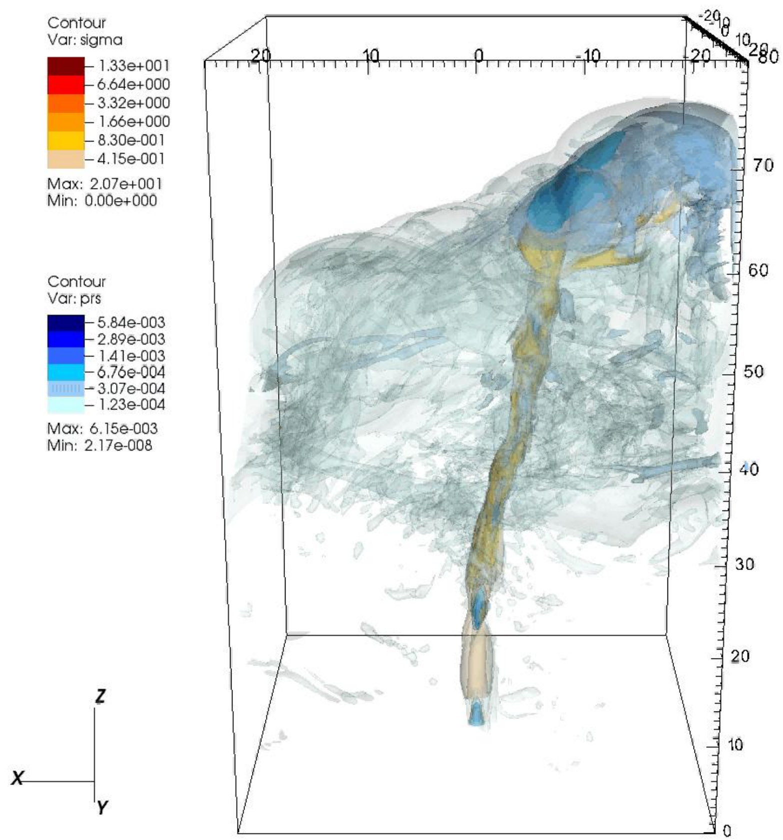
Case	γ	σ
A1	2	0.1
A2	2	1
A3	2	10
B1	4	0.1
B2	4	1
B3	4	10

<ins>*Sigma distribution*</ins>

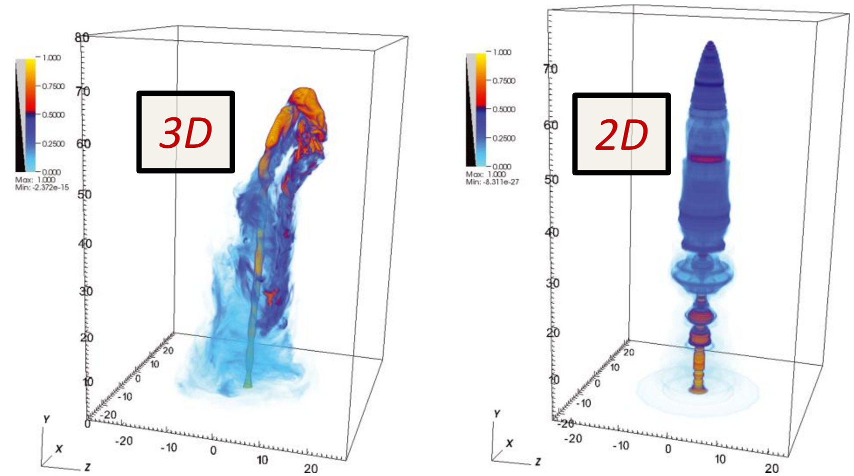


General Features

- Jets have small propagation speed (0.02c – 0.08c);
- Large over-pressurized turbulent cocoons;
- Collimated central spines moving at mildly relativistic speeds;
- Cocoon less magnetized than central spine;
- Large-scale deflections may be present.



General Features



- 3D models very different from 2D counterparts[1]:

- Strong toroidal configurations expected to become unstable to current driven modes. Most unstable mode m=1 (kink);

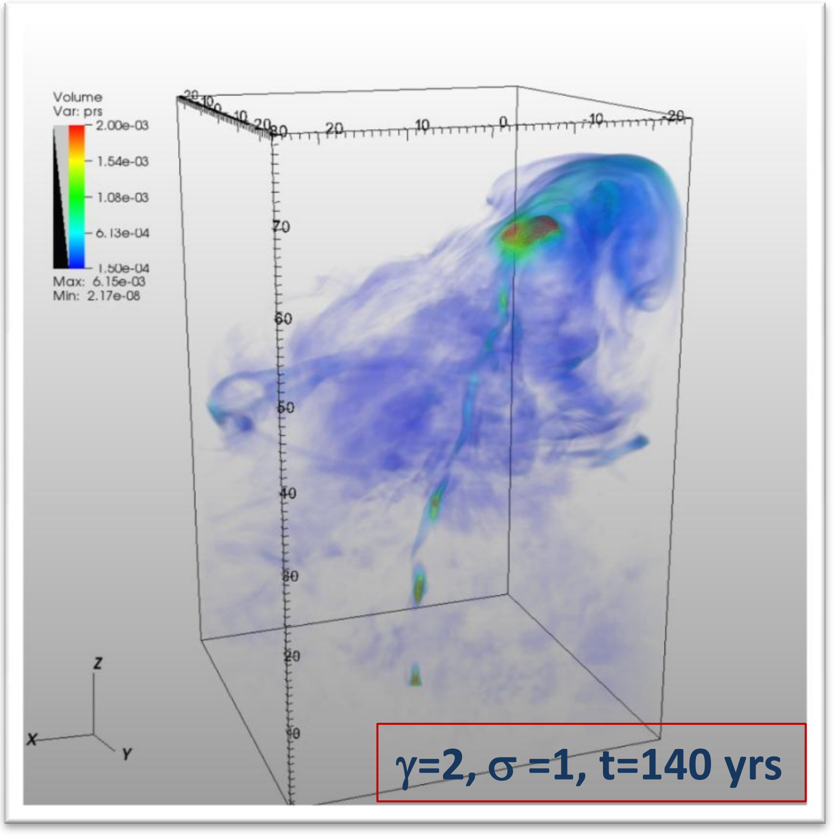
- Jet develops non-axisymmetric structures with large time-dependent deflections off the longitudinal axis;

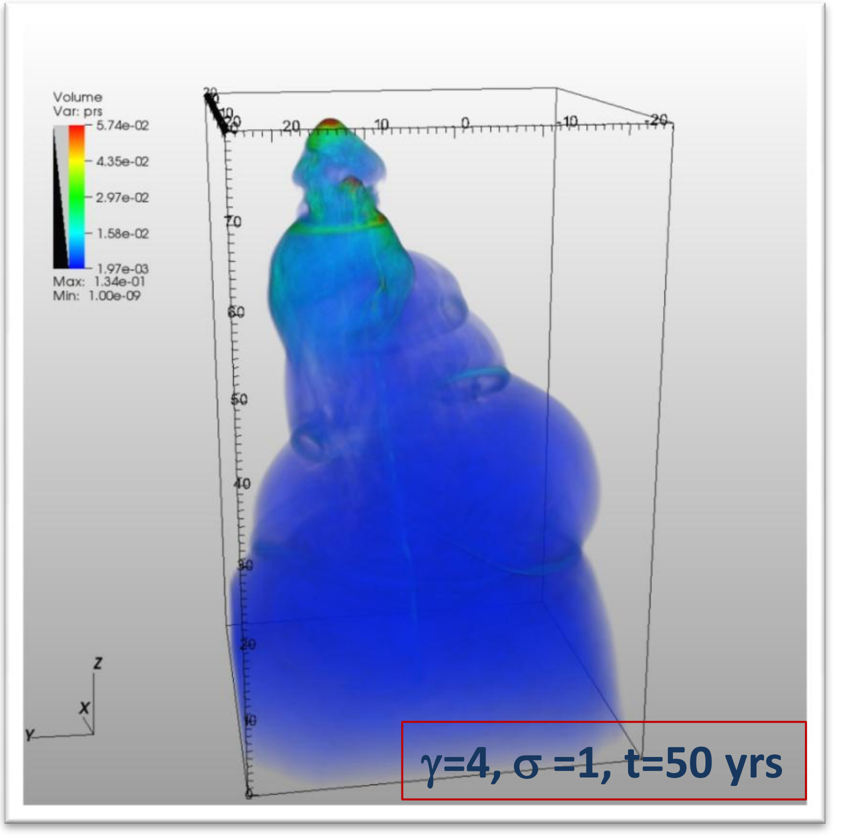
- Deflection time-scale of the order of a few years;

[1]Mignone et al. MNRAS (2010), 402, 7

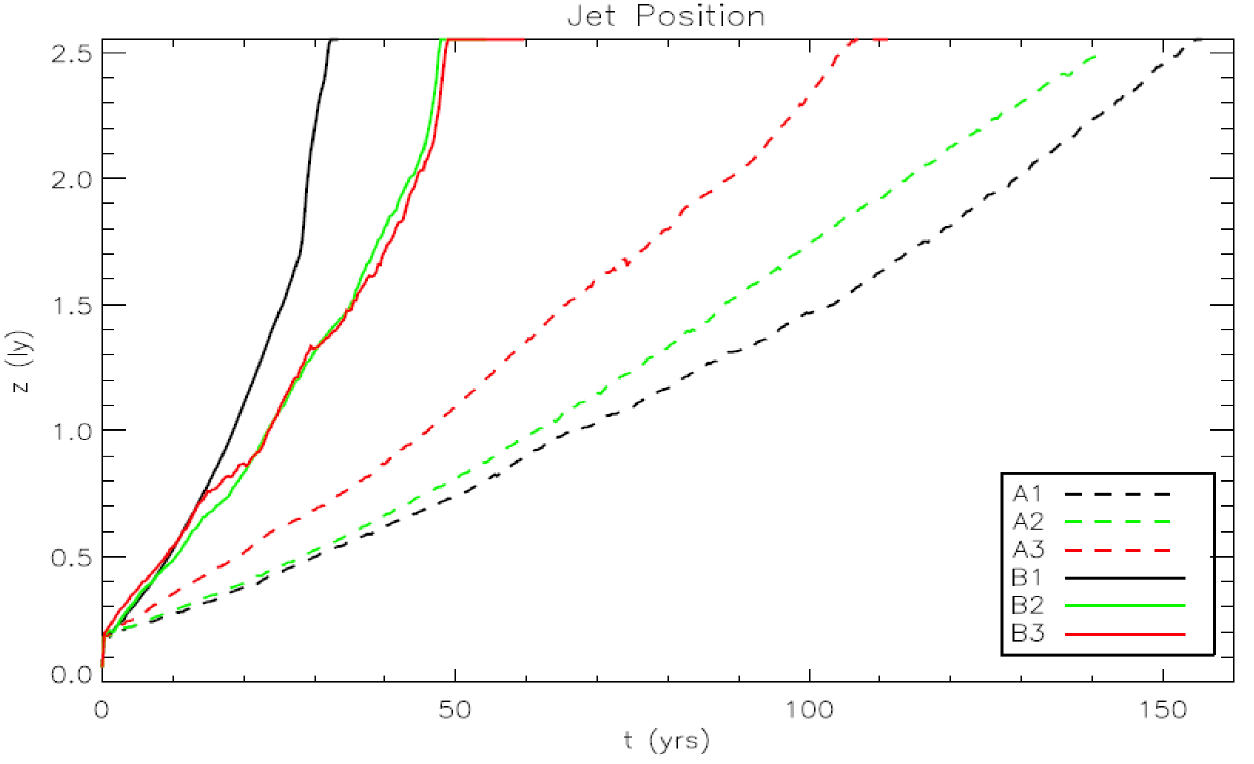
General Features

- Wiggling and deflection more pronounced at the terminal bow shock where magnetic field is amplified:





Jet Position



- Jets are slow because of large density contrast ($\rho_j / \rho_e < 10^{-6}$);
- Faster jets reach the outer edge of the expanding nebula.

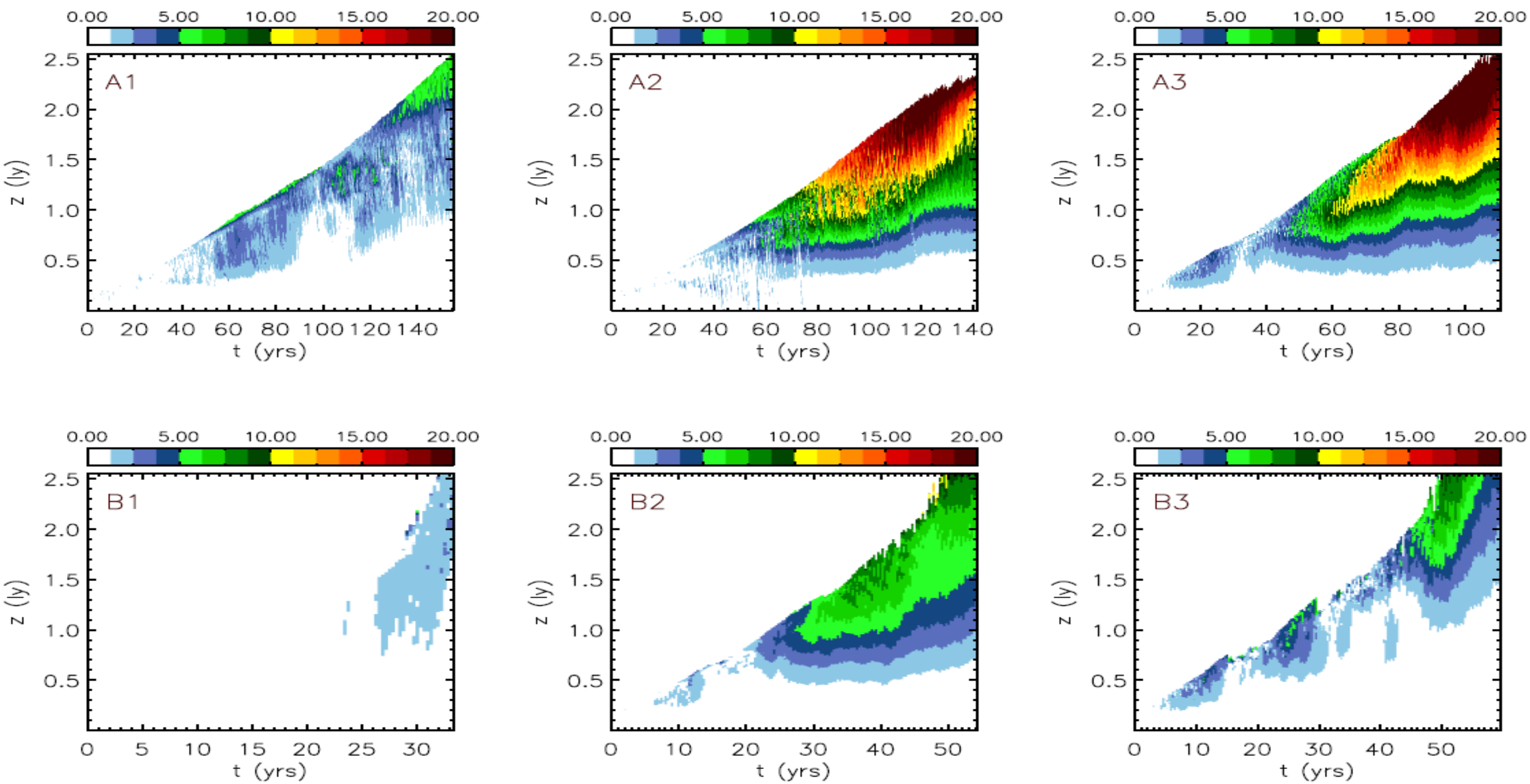
Jet Deflections

- Deflection is quantified using the baricenter:

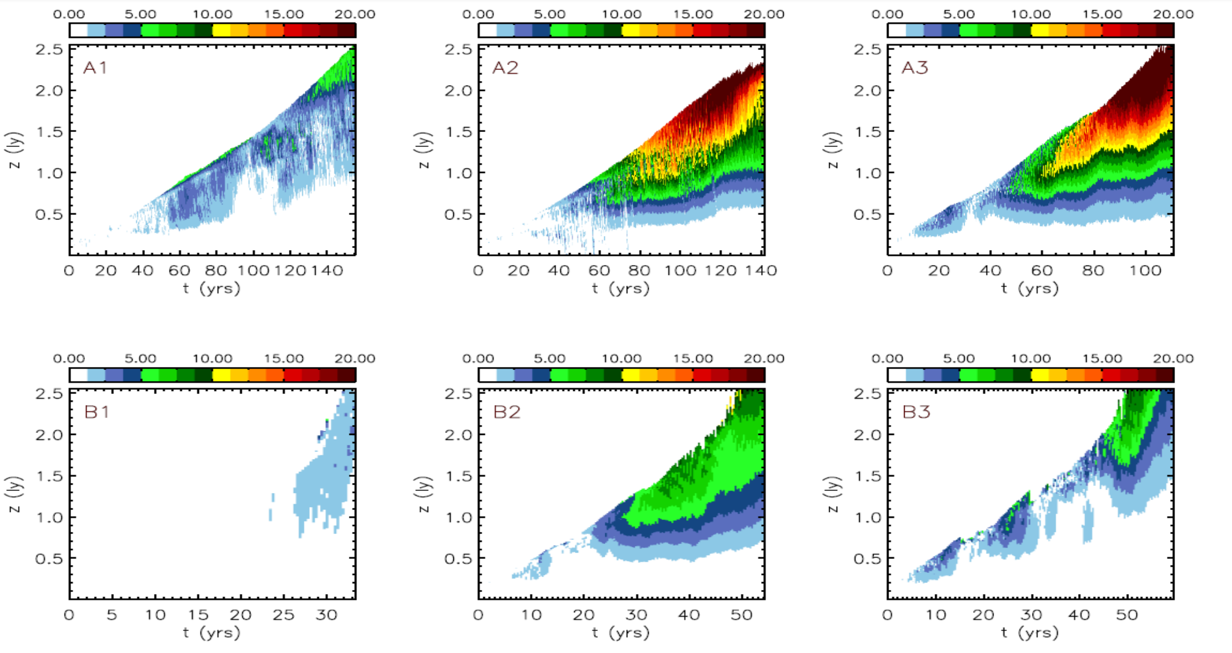
$$\boxed{\bar{R}(z) \equiv \sqrt{\bar{x}^2(z) + \bar{y}^2(z)}}$$

$$\boxed{\bar{x}(z) = \frac{\int xQ(x,y,z)dxdy}{\int Q(x,y,z)dxdy}}$$

$$\boxed{\bar{y}(z) = \frac{\int yQ(x,y,z)dxdy}{\int Q(x,y,z)dxdy}}$$



Jet Deflections

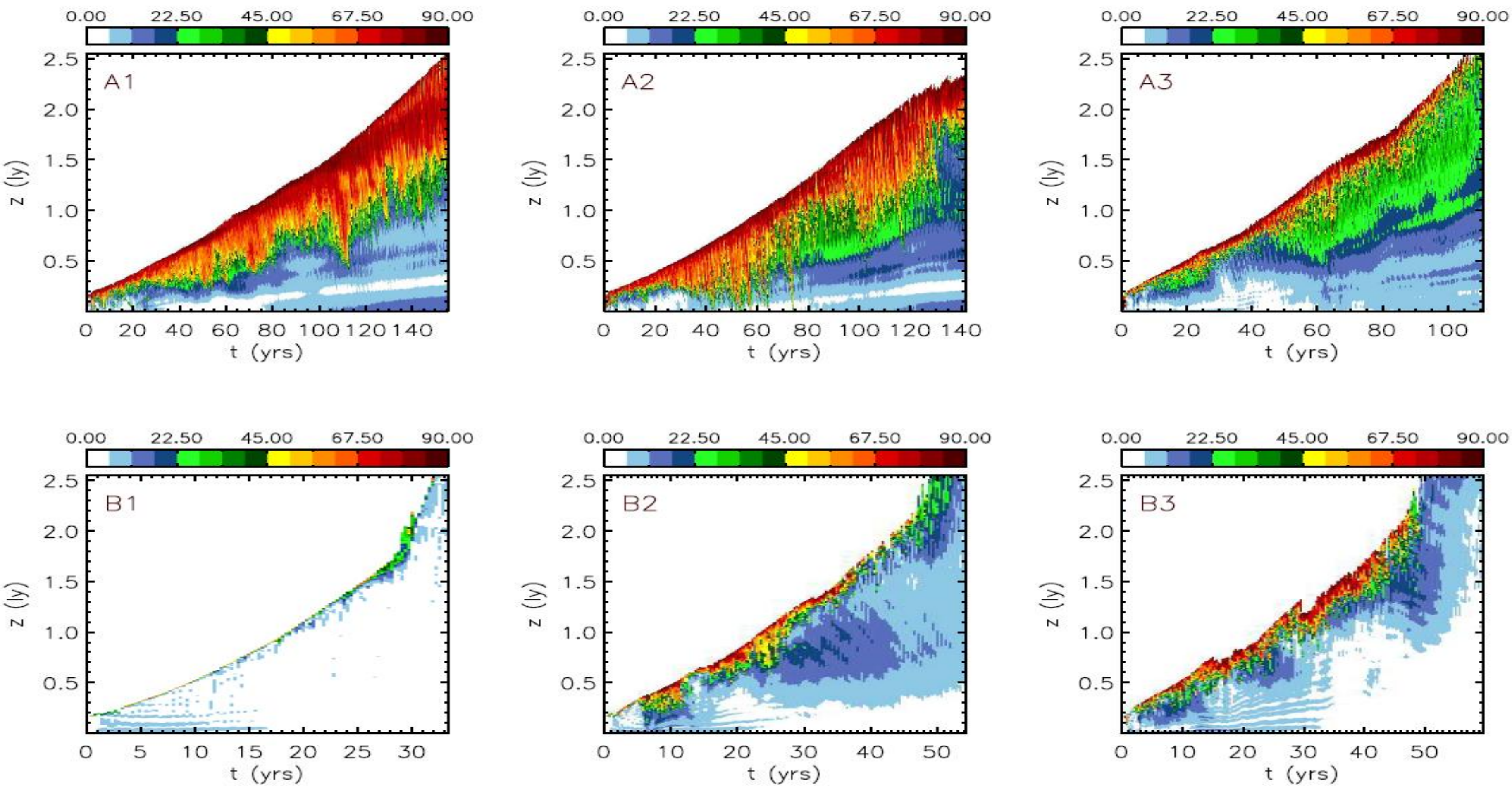


- Case A2 and A3 (low-speed, moderately/highly magnetized) jets show the largest bending (> 20 jet radii);
- Larger Lorentz factors (B2, B3) have a stabilizing effect;
- Weakly magnetized jets (A1, B1) are less affected by the growth of instability;

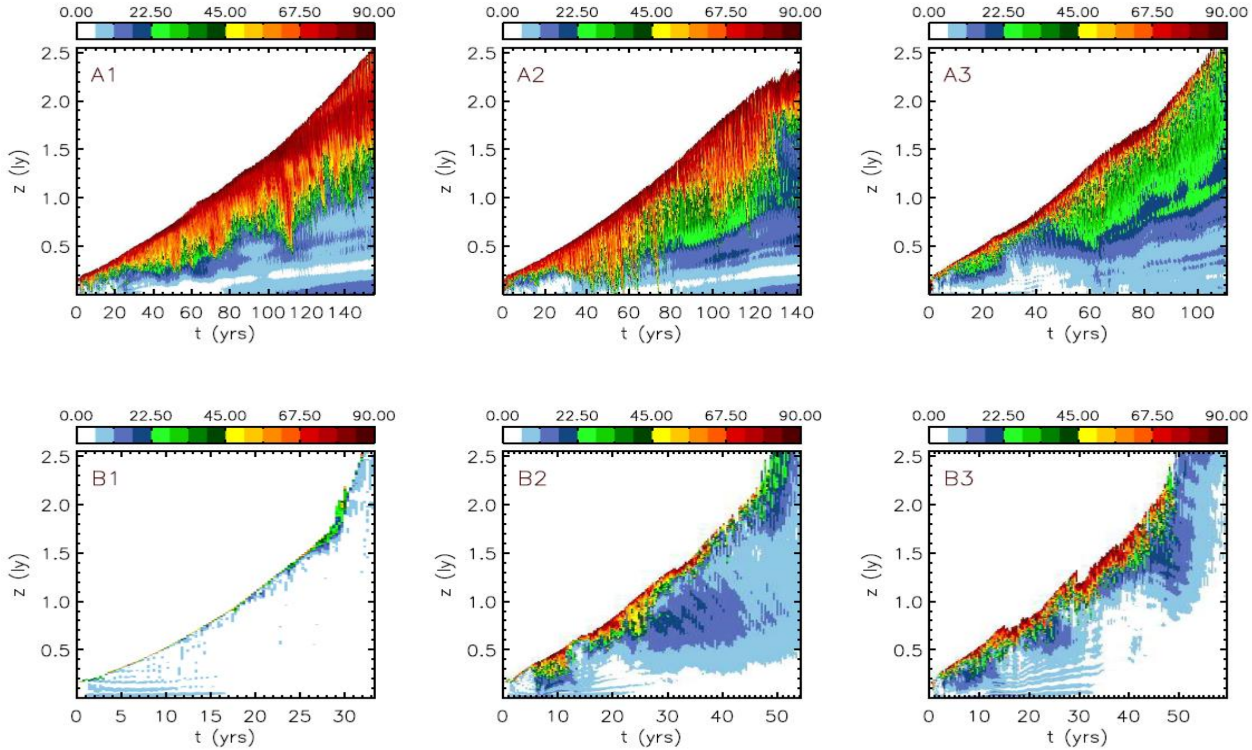
Flow Direction

- Flow direction is measured by computing the average angle of the mass flux vector with vertical direction:

$$\bar{\theta}_{\pm}(z) = \cos^{-1}\left(\frac{\int (v \cdot \hat{z}/|v|)\chi_{\pm}\, dx\, dy}{\int \chi_{\pm}\, dx\, dy}\right)$$



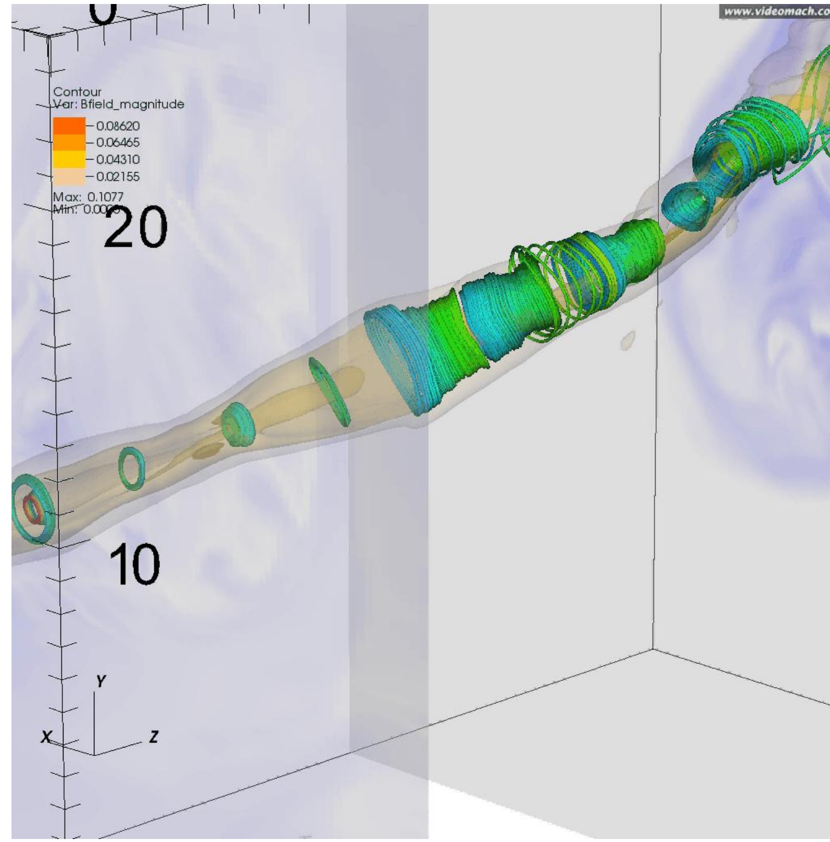
Flow Direction



- Low-speed jets assume a large-scale curved structure;
- High-speed jets more parallel and build kicks in proximity of the jet head;

Magnetic Field

- Magnetic field topology remains mainly toroidal or helical during the propagation;
- Azimuthal field has the effect of “shielding” the core preventing interaction with the surrounding.
- Magnetic field dissipates and becomes turbulent in the cocoon (→ randomization)



Summary

- 3D models of azimuthally confined relativistic jets are very different from 2D axisymmetric models:
  - Kink-unstable non-axisymmetric structures with large time-variability;
  - Large σ ($\gtrsim 1$) leads to considerable jet deflections;
  - Pronounced asymmetric backflows;
  - Jet wiggling progressively more pronounced towards the jet head
  - Multiple strong shocks are formed by change of direction;
- Low-speed ($\gamma \lesssim 2$), moderately/highly magnetized jets ($\sigma \simeq$ 1-10) are promising candidates for explaining the morphology of the Crab jet.
- Future models will consider the jet-torus connection in 3D

Thank you